Class Work Solutions

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$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -7 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -7 \end{array}\right] \xrightarrow{-R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 7 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -7 \end{array}\right] \xrightarrow{-R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 7 \end{array}\right] \xrightarrow{R_1 - 2R_2 \to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -11 \\ 0 & 1 & -2 & 7 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -11 \\ 0 & 1 & -2 & 7 \end{array}\right]$$

Free variable: z. Let $z = t$.

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$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -11 \\ 0 & 1 & -2 & 7 \end{array}\right]$

Free variable: z. Let $z = t$.
Then $x + 3t = -11$ and $y - 2t = 7$.

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#1. $\left[\begin{array}{ccc|c} 3 & 6 & -3 & 9 \\ 2 & 3 & 0 & -1 \end{array}\right] \xrightarrow{\frac{1}{3}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 3 & 0 & -1 \end{array}\right] \xrightarrow{R_2 - 2R_1 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 2 & -7 \end{array}\right] \xrightarrow{-R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 7 \end{array}\right] \xrightarrow{R_1 - 2R_2 \to R_1}$

$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -11 \\ 0 & 1 & -2 & 7 \end{array}\right]$

Free variable: z. Let $z = t$.
Then $x + 3t = -11$ and $y - 2t = 7$.
Therefore

$$x = -3t - 11, y = 2t + 7, z = t$$

is the parameterized solution.

#2.

$$\left[\begin{array}{rrr|r} -2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \end{array}\right]$$

#2.

$$\left[\begin{array}{rrr|r} -2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1 \to R_1} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 1 & -2 & 1 & 3 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} -2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

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$$\left[\begin{array}{rrr|r} -2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1 \to R_1} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2}$$

$$\left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{array}\right] \xrightarrow{\frac{1}{4}R_2 \to R_2} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

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#2.

$$\left[\begin{array}{ccc|c} -2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{array}\right] \xrightarrow{\frac{1}{4}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 + R_2 \to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

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#2.

$$\left[\begin{array}{ccc|c} -2 & 4 & -2 & 2 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 1 & -2 & 1 & 3 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{array}\right] \xrightarrow{\frac{1}{4}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 + R_2 \to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

The leading 1 on the right of the bar indicates that the system has no solution.

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#3. $\left[\begin{array}{cccc|c} 1 & 4 & 1 & 1 & 1 \\ 2 & 8 & -1 & -7 & 8 \\ 2 & 8 & -2 & -10 & 10 \end{array}\right]$

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#3. $$\left[\begin{array}{rrrr|r} 1 & 4 & 1 & 1 & 1 \\ 2 & 8 & -1 & -7 & 8 \\ 2 & 8 & -2 & -10 & 10 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{rrrr|r} 1 & 4 & 0 & -2 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Class Work Solutions

$$\#3. \quad \left[\begin{array}{cccc|c} 1 & 4 & 1 & 1 & 1 \\ 2 & 8 & -1 & -7 & 8 \\ 2 & 8 & -2 & -10 & 10 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 4 & 0 & -2 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

To get this RREF, try doing the following sequence of EROs: $R_2 - 2R_1 \to R_2$, then $R_3 - 2R_1 \to R_3$, then $-\frac{1}{3}R_2 \to R_2$, then $R_1 - R_2 \to R_1$, then $R_3 - 4R_2 \to R_3$.

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#3. $\left[\begin{array}{cccc|c} 1 & 4 & 1 & 1 & 1 \\ 2 & 8 & -1 & -7 & 8 \\ 2 & 8 & -2 & -10 & 10 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 4 & 0 & -2 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

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The variables y and w are free variables. Set $y = s$ and $w = t$.

#3. $\left[\begin{array}{cccc|c} 1 & 4 & 1 & 1 & 1 \\ 2 & 8 & -1 & -7 & 8 \\ 2 & 8 & -2 & -10 & 10 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 4 & 0 & -2 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

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The variables y and w are free variables. Set $y = s$ and $w = t$.

Then $x = -4s + 2t + 3$ and $z = -3t - 2$.

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#3. $\left[\begin{array}{cccc|c} 1 & 4 & 1 & 1 & 1 \\ 2 & 8 & -1 & -7 & 8 \\ 2 & 8 & -2 & -10 & 10 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 4 & 0 & -2 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

To get this RREF, try doing the following sequence of EROs: $R_2 - 2R_1 \to R_2$, then $R_3 - 2R_1 \to R_3$, then $-\frac{1}{3}R_2 \to R_2$, then $R_1 - R_2 \to R_1$, then $R_3 - 4R_2 \to R_3$.

The variables y and w are free variables. Set $y = s$ and $w = t$.

Then $x = -4s + 2t + 3$ and $z = -3t - 2$.

The parameterized solution is therefore

$$x = -4s + 2t + 3, \quad y = s, \quad z = -3t - 2, \quad w = t.$$